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Sources of Concepts: A Cultural-Developmental Perspective

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Piaget's legacy is manifest across a wide range of research endeavors in the behavioral and social sciences. In this chapter, I point to the way that Piaget's theory provides an important basis for my own work on the interplay between culture and cognitive development. In keeping with my charge by the volume's editors, I target sources of development.

INTRODUCTION

Researchers seeking to explain cognitive development have looked to a range of possible social and cultural sources. Some have analyzed social interactional processes, either between adults and children (e.g., Wertsch, 1979; Wood, Bruner, & Ross, 1976) or between peers (e.g., Saxe, Gearhart, Note, & Paduano, 1993; Webb, 1982) as sources of development. Others have identified language and other symbolic or material artifacts as sources of change (Miura, Okamoto, Kim, Steer, & Fayol, 1993). Still others have identified sources in the structures of practices in which children are participants (Berry, 1966; Price-Williams, Gordon, & Ramirez, 1969).

What perhaps best distinguishes Piaget's developmental theory from accounts that locate sources of change in factors *external* to the child is Piaget's strong emphasis on an *internal* locus of causation. Rather than external cultural forces, Piaget's work emphasized processes of self-regulated change as fundamental to the analysis of source. Indeed, in a certain

sense, the notion of an external source is a contradiction in terms for Piagetian theory. Although external factors might have implications for change, ultimately the individual, or Piaget's "epistemic subject," had primacy in effecting change and thus was the root source of development (Piaget, 1952, 1977).

In this chapter, I outline an approach for analyzing sources of conceptual development linked to cultural practices. Underlying my remarks is an argument that a systematic treatment of source requires an incorporation of both culture and individual agency in a single analytic framework. Readers knowledgeable in Piagetian theory can find the organizing theme of the volume—Piaget's legacy—well represented. Key Piagetian ideas of epigenesis (that new structures of knowledge have their roots in previous structures) and construction (that individuals are active agents in the epigenetic process) are central to the treatment of source. At the same time, the approach departs from Piagetian thought: Piaget's key focus on cognitive operations that are universal and developed independently of particular cultural practices does not afford a differentiated treatment of history and culture in treatments of cognitive development. Such a concern with historical and cultural processes in a treatment of sources of development is the principal focus here.

The approach that I sketch is *cultural* in that everyday practices—recurrent socially organized activities—are targeted for analysis. I build on research on learning in such practices as games (Fall, 1997; Guberman & Rahm, 1996; Iwanaga, 1997; Nasir, 1997), entrepreneurial activities such as street selling (Carraher, Carraher, & Schliemann, 1985; Saxe, 1991) or weaving (Greenfield, in press; Saxe & Gearhart, 1990), and classroom practices linked to schools (e.g., Ball, 1993; Cobb, Wood, & Yackel, 1993; Gearhart et al., 1997; Lampert, 1990). In the treatment of cognition in practice, individual activity and the social organization of practices are understood *reflexively*: Children's goal-directed activities are in themselves aspects of practices, and at the same time practices give form and social meaning to children's ongoing goal-directed activities. In structuring and accomplishing goals, individuals create novel cognitive developments at once linked to social life and to their own constructive efforts.

The approach is also *developmental*. Cognition in practices (as well as practices themselves) is understood as a process undergoing transformation. The analytic focus is on three related kinds of cognitive change linked to practices: the short-term formation of representations and strategies in individuals' efforts to accomplish goals that emerge in practices (microgenesis); developmental shifts in the structure of individuals' repeated efforts to create and accomplish recurrent goals in practices (ontogenesis); the spread or diffusion of means to accomplish emergent goals in practices in communities (sociogenesis).

THE PRACTICE OF CANDY SELLING

To illustrate the cultural-developmental framework as it bears on questions of sources of development, I draw on observations and analyses of 6- to 15-year-old boys engaged in the practice of candy selling in northeastern Brazil. The children were poor, and many had little or no schooling. My focus is on the unusual mathematics that sellers created as they plied their trade and on what these observations and analyses can tell us about sources of children's mathematics.

In the context of Brazil's long inflationary economic history, the sellers in my study dealt with very large currency values in their street mathematics. The Brazilian unit of currency at the time was the cruzeiro, and tens of thousands of cruzeiros passed through sellers' hands in a day's business. In observations of sellers in their practice and in interviews with them on a range of mathematical tasks, I found that the math that they created in their activities had little resemblance to the mathematics that children learned in school (Saxe, 1991). Indeed, sellers' principal representational system for large numerical values was the currency system. They identified coins and bills from 50 through 50000 cruzeiros, not by the numerals printed on them, but by their figurative characteristics (e.g., color, portraits) and translated these currency units into verbal representations for quantity. Furthermore, they did not use a written arithmetic for performing arithmetical computations, but instead often performed arithmetic computations by using procedures that employed currency: tallying physical quantities of bills or mentally adding and subtracting currency-linked linguistic representations of quantity. They also computed complex markup computations and adjusted for inflation in these computations by using procedures that were widespread in their practice but not known to children in school.

To understand how sellers' mathematics took form in their practice, I focused on sellers' emergent goals as they plied their trade and on developmental transformations in the character of sellers' mathematical problem solving and understandings.

Children's Emergent Goals in the Candy-Selling Practice

Candy sellers created and accomplished a wide variety of goals in their practice. Sometimes, these goals were social and involved issues of self-protection, as when a child tried to affiliate with a group of sellers that traded in one neighborhood to protect himself from "crabs" (sellers who crossed territorial boundaries) or to protect himself from being labeled a crab himself. Other times, goals were economic, as when a seller attempted to bargain with a customer. Still other times, goals involved mathematics,

as when a seller was engaged in a markup computation and tried to determine how much he should sell his candy for by units if he paid 12500 cruzeiros in the store and wanted to make a good profit. In the analyses to follow, I show how sellers' mathematical goals emerged and took form in relation to the four parameters depicted in Fig. 11.1: the activity structure of the practice, patterns of social interaction that took form in the practice, artifacts or conventions that were valued in the practice, and the understandings that children brought to bear on practices. As sellers structured and accomplished goals in practices, their developing mathematics became interwoven with their own constructive efforts as well as with social and cultural dimensions of their daily lives.

Activity Structures

Candy selling is an entrepreneurial practice. Typically, children's principal motive for participation is economic. To accomplish their practice, sellers purchased their boxes from wholesale stores during a purchase phase, priced their candy for sale in a prepare-to-sell phase, sold their candy in the street in a sell phase, and then selected new wholesale boxes for purchase in a prepare-to-purchase phase. The cycle then returned to the purchase phase. This economic motive often colored sellers' activities, including their mathematical goals. The inner rectangle of Fig. 11.2 depicts this entrepreneurial structure of activity that takes form in the candy selling practice. To make ends meet in their entrepreneurial activity, sellers had to structure and accomplish various mathematical goals involving the rep-

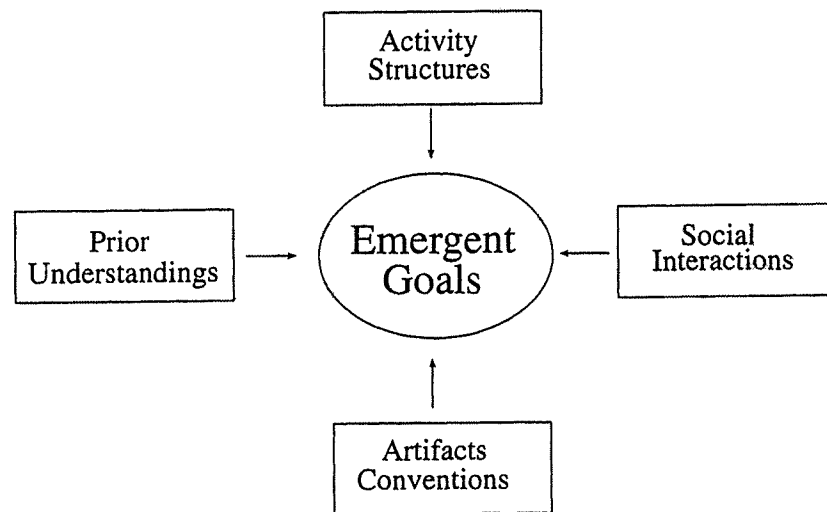


FIG. 11.1. Candy sellers' mathematical goals emerged in relation to the activity structure of the practice, valued artifacts and conventions, social interactions, and sellers' own prior understandings.

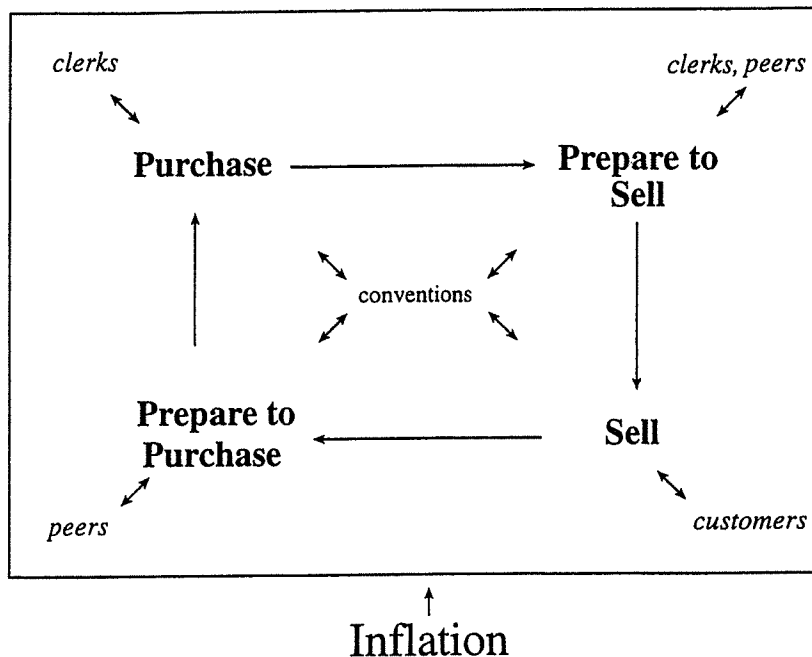


FIG. 11.2. A schematic of the candy selling practice.

resentation of numerical values, the addition and subtraction of values, the comparison of ratios, and goals linked to markup from wholesale to retail price.

Artifacts and Conventions

Various artifacts and conventions that are valued by sellers entered into the mathematical goals that emerged as children plied their trade (represented in the center of Fig. 11.2). Consider both the artifact of currency and a pricing convention that has emerged over the history of the practice.

Because of a long history of inflation, the value of currency in use in selling was quite high. For example, boxes cost thousands of cruzeiros each in wholesale stores, and bills in common use in transactions included notes of Cr\$200, Cr\$500, Cr\$1000, Cr\$5000, Cr\$10000, and Cr\$50000. As a result, when sellers constructed mathematical goals in their practice—whether purchasing a box in a wholesale store or in a transaction with a customer—the goals often involved numbers of great magnitudes.

The price-ratio convention figured centrally into sellers' emergent goals. In selling, virtually all sellers observed offered their candy to customers in the form of a price ratio of units of candy to a particular bill value, generally for some number of units for Cr\$1000. Older sellers typically offered their candy to customers for two prices—three packages for Cr\$500 or five for Cr\$1000. The price ratio became implicated in sellers' mathematical goals

in a number of ways. The convention reduced the complexity of arithmetic goals that might emerge in making change. For instance, a seller simply exchanged three bars for a very common unit of currency, the Cr\$1000 note, and often no change was required in such transactions. The convention, however, also led sellers to become engaged in ratio comparisons and with other more complex mathematics: In their markup computations, for example, a seller translated a multiunit wholesale box price (e.g., Cr\$12000) in terms of a retail price ratio (e.g., X units for Cr\$1000) such that he ended with an appropriate profit margin. Whether in face-to-face exchanges with customers or in markup computations, the price-ratio convention was intrinsically linked to the mathematical goals that emerged for sellers.

Finally, another pricing convention was widely used for determining an appropriate profit margin in the selling practice. This convention was called *meio-pelo-meio*. As explained by older sellers, the convention prescribed that one should set a price so that one half the profit goes for the purchase of the next box and one half is kept by the seller as profit. By using this convention, sellers' markup goals took a particular form. One feature of this convention was related to sellers' mathematical goals: Sellers (often unknowingly) adjusted their selling price for inflation when using the convention, because the profit margin always remained 100%, regardless of an increase in wholesale prices for candy boxes.

Social Interactions

At each phase in the structure of the practice, sellers typically interacted with other people, whether clerks, customers, or peers (represented in the inner corners of Fig. 11.2). These social interactions that emerged in a practice simplified the construction and accomplishment of some goals and complicated others. For example, in the prepare-to-sell phase, having purchased a box of candy containing typically 30, 50, or 100 units for which they paid thousands of cruzeiros, sellers had to determine an appropriate price for retail sale. The arithmetic goals that emerged in producing markups differed for sellers of different ages. Particularly with young sellers, clerks or older peers told children an appropriate retail price and accomplished the markup computation for the seller; at other times, sellers marked up their candy in the street, when children provided assistance to and negotiated with each other about appropriate markup. At still other times, sellers marked up prices on their own. Thus, depending on social supports for markup, the mathematical goals that sellers constructed to accomplish markup varied: For the older seller, markup goals and means of achieving them could be quite complex; for a younger child, a markup goal would not involve mathematics.

Bargaining transactions are yet another example of the way that social interactions afforded the emergence of particular goals. In bargaining, a customer indicated that the cost of the candy was too high (or perhaps that the wrapper was sticky owing to the summer's heat). Ensuing negotiations sometimes gave rise to sellers' needs to decide whether a revised price was viable in the context of profit concerns. Like the construction of markup goals, there was evidence in my study that the complexity of goals that emerged in bargaining varied with sellers' ages. Young sellers typically sold for a fixed price that they were told to use, whereas older sellers more typically adjusted prices in bargaining transactions.

Prior Understandings

The mathematical understandings that sellers brought to bear on the practice—the fourth parameter in Fig. 11.1—were fundamental to the mathematical goals that emerged in a practice. Whether we consider the character of sellers' representational, arithmetic, comparison, or markup goals, in each domain, goals took form in relation to children's mathematical understandings. The means and subgoals that sellers elaborated in each domain depended on these understandings. For instance, a seller who did not understand the relation between wholesale price and retail price did not generate markup goals; or if he did, the mathematical goals in his computation were quite different from the goals of the seller who did.

Sellers' mathematical goals became constituted as they plied their trade. They took form in relation to activity structures, artifacts, social interactions, and prior understandings. In structuring and accomplishing these goals, sellers' mathematics became rooted in both culture and individuals' constructive activities.

Development in the Candy-Selling Practice

In the following analysis of development, I focus on the interplay in development between cultural forms such as currency or a number system and the functions that these forms serve for individuals in their practices. The approach draws on aspects of Werner and Kaplan's (1962) treatment of form-function relations in the development of language, a treatment that I subsequently extended in my own work on cultural practices and children's mathematics (Saxe, 1991, 1996; Saxe, Guberman, & Gearhart, 1987). The formulation also reflects a dominant concern of Vygotsky and other Soviet writers (e.g., Leontiev, 1981; Vygotsky, 1978, 1986), who were interested in identifying analytic units that preserved intrinsic relations between cognitive and sociocultural processes in analyses of development.

Cultural forms vary in the way they afford or are specialized to serve particular functions in practices. For instance, in the domain of arithmetic, the Hindu-Arabic numeration system is highly specialized for computation. Other number systems, like the 27-body-part counting system of the Oksapmin of Papua New Guinea (Saxe, 1982), have been specialized for serving cognitive functions like counting and certain forms of measurement, but not for arithmetic computation. Still other artifacts, such as currency systems, have been specialized for economic exchanges. Next, I target three kinds of development, each of which entails an interplay between form and function in mathematical activities.

Microgenesis

Although forms have evolved over social histories to serve particular functions in practices, in activity, individuals adapt or repurpose forms to accomplish their own goals. Microgenesis¹ refers to this transformative process whereby individuals tailor cultural forms to become means to accomplish goals in practices. To illustrate, consider a seller's activity in the prepare-to-sell phase.

Ten-year-old Luciano was engaged with a markup problem. He paid Cr\$7000 at the wholesale store for his 30-unit box of candy bars, and he needed to determine how much to sell the candy for in the street so that his candy moved quickly and he made a good profit. Luciano's approach, like many other sellers, was to first focus on a determination of the price that the box would bring if it were sold at a particular price ratio. He began by considering a price ratio of three bars for Cr\$1000. To create a means for accomplishing his computation, he emptied his box on the ground. Then, he replaced bars in the box in groups (see Fig. 11.3). With each placement, he created many-to-one correspondences between the cash price of a Cr\$1000 bill and groups of three bars, and incremented his running total by Cr\$1000 with each placement. In the end, Luciano compared his calculated street price (Cr\$10000) with the wholesale price (Cr\$7000) and made use of the *meio-pelo-meio* (half-half) convention as a norm to determine the adequacy of his profit. If he felt he would not earn

¹Some researchers have made use of *microgenesis* to refer to a methodological approach involving the intensive study of shifts in children's strategies and/or cognitive structures over short periods (see, for example, Siegler & Jenkins, 1989). My use of the term is more consistent with earlier treatments of the construct (Vygotsky, 1986; Werner & Kaplan, 1962), in which the very process of schematization of a phenomenon, perceptually or conceptually, is understood as a short-term developmental process. As conceptualized in the present discussion, microgenesis is neither a methodological approach nor a small-scale version of ontogenetic change. Rather, it is a process in which forms with the cognitive functions they afford are transformed into means for accomplishing emerging goals.

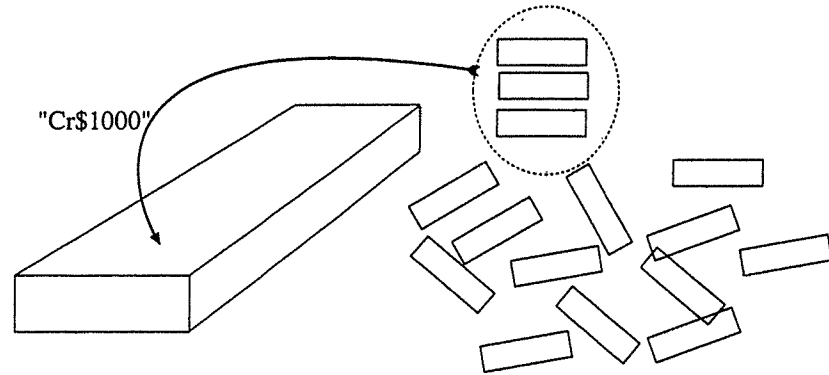


FIG. 11.3. Luciano replaced candies in his box in groups of three, adding "one thousand" to a running total with each placement, until all candies were replaced in his box. This procedure allowed Luciano to determine the gross value of the box, if the price ratio of 3 candies for Cr\$1000 was used in sales.

enough profit, he repeated the process with a different grouping and then compared the two results. The method was laborious but reliable.

The three forms with which Luciano worked in his activity—the price-ratio form, the currency, and the candy itself—have emerged over the history of a range of practices to serve culturally valued functions. Commercially packaged candy served as a commodity for trade and consumption; currency served as a common unit of exchange; the price ratio served to mediate sales exchanges between seller and customer. These functions were not fixed. Indeed, in Luciano's markup activity, he adapted and repurposed these forms to achieve his markup goal.

Two features are noteworthy about the microgenesis of these forms into means for accomplishing markup. First, Luciano schematized these forms as mathematical entities and adapted them to serve mathematical functions. Thus, Luciano conceptualized the candy and currency in a relation of many-to-one correspondence—three bars to 1 Cr\$1000 bill; then he added Cr\$1000 to a running total for each many-to-one correspondence. Thus, in Luciano's activity, these forms became structured as mathematical entities and served mathematical functions.

Second, Luciano deployed the materials strategically. Making use of the properties of candy bars as discrete and movable objects, Luciano emptied the box and replaced the candy in groups of three until none were left on the ground. Thus, he used the materials themselves as an "external" memory to "tell" him when he had exhausted all potential sales. Through this strategy, Luciano insured that no candy was treated as unsold in his summation. In the microgenesis of Luciano's solution, currency and candy became at once the mathematical and physical means to accomplish markup

goals, functions that were not given but that emerged in the microgenesis in Luciano's markup.

Ontogenesis

Gaining insight into the development of sellers' mathematics requires an analysis of the epigenesis of sellers' knowledge: Can we identify the origins of 10-year-old Luciano's strategy in younger children's engagement with the practice? Further, can Luciano's strategy be a seed for subsequent, more sophisticated efforts in his own prospective development? To address these questions, I consider first a seller who is younger than Luciano and his use of the candy and price-ratio forms in his activities; then, I turn to a seller older than Luciano and consider the way that Luciano's strategic efforts may serve as seeds for subsequent ontogenetic shifts linked to the practice.

Wilher at 6 Years. In the street, 6-year-old Wilher was observed selling his candy at five packages of lifesavers for Cr\$1000. For this 6-year-old seller, the price-ratio form served the cognitive function of mediating exchanges of candy for currency in seller-customer transactions: Wilher exchanged, for instance, one Cr\$1000 bill for five candy packs over and over again (as depicted in Fig. 11.4). Thus, like 10-year-old Luciano, Wilher was structuring the price-ratio form as a means of accomplishing an emerging goal, although the means was organized to serve a very different function from markup computation. In fact, for Wilher, issues of price markup were taken care of by others, such as store clerks, older peers, or relatives. Wilher's use of the price ratio—the activity of engaging in the many-to-one exchanges of bills for candy—may well be a critical seed for the kind of strategy we observed with Luciano. Indeed, Luciano appeared to have appropriated the price-ratio form from seller-customer transactions and used it to serve a new function—as a means to refer to potential transactions for the purpose of determining markup.

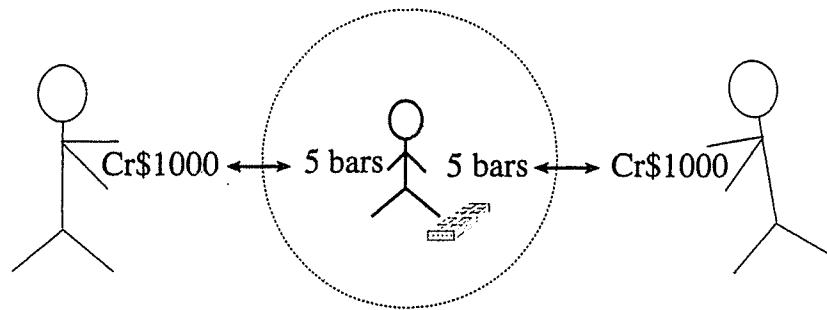


FIG. 11.4. In selling with the price-ratio convention, Wilher creates a series of many-to-one correspondences as he exchanges five candies for each one thousand cruzeiro bill.

Antonio at 14 Years. When we consider a seller older than Luciano, we find evidence that Luciano's strategic efforts at 10 may be the seeds for later shifts in markup strategies. Antonio began his day with a full box of candy bars. The box contained 30 units, and he paid Cr\$8000 for the box. He sold the bars at three for Cr\$1000. When questioned about how he determined his prices, Antonio explained that he counted each group of three as Cr\$1000; each count of three represented one sale, and he counted two groups of three at a time as depicted in Fig. 11.5. Thus, he counted the gross price by stating: "These two (two groups of three) bring Cr\$2000; these two (two groups of three) Cr\$4000, these two, Cr\$6000; . . . these two, Cr\$10000. Antonio determined that selling the units for three for Cr\$1000 on the street yielded a gross of Cr\$10000. He then subtracted the wholesale price of \$8000 from the Cr\$10000 and determined that he would net Cr\$2000. Antonio's calculation as depicted in Fig. 11.5 showed a condensed and abbreviated use of the price ratio to speed computation.

An Epigenetic Sequence. The cross-section of development at 6, 10, and 14 years provides evidence of an epigenetic sequence in which a seller's previous strategic efforts linked to practice participation provided a basis for him to construct new knowledge. In this process, a seller repurposed cultural forms initially used to serve one function as he generated new goals in the course of participating in practices. Thus, the young seller first used the price-ratio form to serve the function of mediating seller-customer exchanges, and he knew that issues of markup would be taken care of by others. The child created many-to-one correspondence between a currency unit—the Cr\$1000 bill—and five units of candy. Later, as in the case of 10-year-old Luciano, the seller reconstructed the purpose of

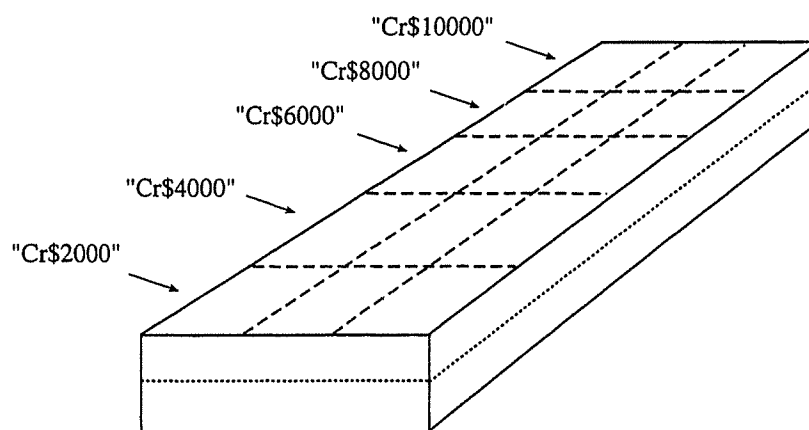


FIG. 11.5. A depiction of Antonio's abbreviated procedure for determining the gross value of his box if the price ratio of 3 candies for Cr\$1000 were used in sales.

the price ratio. Luciano used the price ratio referentially: He deployed the ratio as a way to represent potential customer–seller transactions. In his markup computation, he emptied the box on the street and schematized the ratio as a means of representing potential customer–seller transactions. Thus the many-to-one correspondence which had formerly occurred between child and customer was now used to refer to hypothetical customer–seller transactions. This second order use of the ratio allowed the seller to calibrate and recalibrate potential gross prices until he found an appropriate match by using his *meio-pelo-meio* convention as a guide (half the profit for the next box and half for the seller). In the case of the oldest seller, 14-year-old Antonio, we see an abbreviation of means of the hypothetical correspondence between seller and customer. Here, the price-ratio form was hardly discernible in his strategic approach and took the form of a count: “Cr\$2000, Cr\$4000, Cr\$6000, Cr\$8000, Cr\$10000,” in a very rapid street price computation.

Sociogenesis

Sociogenesis involves the spread and evolution of means for solving and accomplishing goals. As individuals adapt and repurpose forms into means in practices, these means may be imitated and valued by others to accomplish practice-linked goals. The appropriation and gradual institutionalization of means in a community is central to the dynamics that lead to new cultural forms.

Consider the price ratio as a valued cultural form. In its early genesis in the selling practice, I suspect that older sellers who sold their candy in single units tried using discounting conventions used in other practices—selling a greater volume for a discounted price. The price ratio may then have spread to other sellers and became valued perhaps because it reduced the need for change, served as a convenient way to offer customers a bargain, and possibly increased volume of sales. In addition, and perhaps later, the ratio pricing began serving additional functions. For instance, it became a convenient way of accommodating for inflating prices by lowering numbers of units while retaining the principal currency unit. It also became a popular means of computing markup, as we have seen in the cases of Luciano and Antonio.

The circle is then complete: In individuals' appropriation of forms to accomplish emerging goals in practices, they adapted and repurposed earlier sociogenetic developments through microgenetic processes and built on earlier constructions in ontogenesis. In turn, the microgenetic transformation of forms into means in activity served as a basis for new processes of sociogenesis involving the spread and institutionalization of new cultural forms. Such an interplay between developmental processes pervades cul-

tural practices and roots individual agency and culture in the same dynamic system.

CONCLUDING REMARKS

In various publications, Piaget cited cross-cultural differences in rates of development through his stages as indicators that development was not simply a product of a maturational unfolding. The interplay between sociohistorical and cognitive developmental processes, however, was never a central target of his genetic epistemology, which invariably focused on cognitive structure and principles of self-regulation without a coordinated treatment of practice. Even Piaget's early seminal works that touched on these issues, *The Moral Judgment of the Child* (1932) and *Play, Dreams, and Imitation* (1951), did not elaborate analytic methods for understanding children's constitutive role in the structure of practices and the way that practices in turn frame developmental constructions.

In my brief sketch of a cultural-developmental framework, I have pointed to the importance of integrating accounts of practice and of development in treatments of microgenesis, ontogenesis, and sociogenesis. Such an approach creates needed analytic room for key Piagetian themes like agency and epigenesis in accounts of the socially and historically situated aspects of knowledge. Indeed, an integration of these two tacks—analyses of cultural practice and development—is essential for a systematic treatment of sources of concepts.

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